THEORETICAL STUDY OF THE ELECTROWEAK INTERACTION – PRESENT AND FUTURE[1]

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Abstract: There have been several important recent developments in precision electroweak tests. These include: the new LEP energy scan during the 1993 run; the first high-precision results on the left-right asymmetry from the SLD Collaboration at SLAC; the probable discovery of the top quark by the CDF Collaboration at Fermilab and the determination of its mass. I will discuss the implications of these and earlier results for testing the standard model; for the standard model parameters, including the top quark mass, the Higgs mass, the weak mixing angle, and the strong coupling constant, α_s; and the search for new physics. In particular, given the CDF direct determination of m_t it is now possible to severely constrain certain types of new physics by separating the contribution from new physics from the dependence on m_t.

1 Introduction

- The Two Paths: Unification or Compositeness

- Recent Data

- Radiative Corrections

- Results: m_t, M_H, α_s, $\sin^2 \theta_W$

- New Physics

2 The Two Paths: Unification or Compositeness

Most work in particle physics today is directed towards searching for the new physics beyond the standard model. Although there are many theoretical ideas for the nature of such new physics most possibilities fall into one of two general categories.

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[1]The results presented here are based in part on an analysis in collaboration with Jens Erler, in progress.

The first, which I describe as the Bang scenario, involves the unification of the interactions. In such schemes there is generally a grand desert up to a grand unification (GUT) or Planck scale (M_P). This is the natural domain of elementary Higgs fields, supersymmetry, GUTs, and superstring theories. If nature should choose this route there is a possibility of probing to M_P and to the very early universe. There are hints from coupling constant unification that this may be the correct path. Some of the implications are that there should be supersymmetry, which can ultimately be probed by finding the new superpartners at the LHC. Secondly, one expects to have a light Higgs boson, which acts much like the standard model Higgs except that it must be lighter than $110 - 150$ GeV, which should be detectable at the LHC or possibly at LEP 2. (The standard model Higgs could be as heavy as $600 - 1000$ GeV.) Finally, a very important prediction of at least the simplest cases is that one expects an *absence* of deviations from the standard model predictions for precision electroweak tests, CP violation, or rare K decays, because of the decoupling of the heavy superpartners. Of course, it is hard to take the observed absence of such deviations as compelling evidence for supersymmetric unification, but they are nevertheless suggestive. Some such schemes also lead to predictions for m_b, proton decay, neutrino masses, and occasionally rare decays.

If the coupling constant unification is not just an accident there are very few new types of physics other than supersymmetry that could be present without spoiling it (unless two new effects cancel). These include additional heavy Z' bosons, gauge singlets, and a small number of new sequential, mirror, or exotic fermion families.

The other general possibility is the Whimper scenario, in which nature consists of onion-like layers of matter at shorter and shorter distance scales. This is the domain of composite fermions and scalars and of dynamical symmetry breaking. Experimental limits imply that any new layer of compositeness would have to be strong binding, and is therefore not analogous to previously observed levels of compositeness. If nature should choose this route, then at most one more layer would be accessible to us at the LHC and future colliders. Such schemes generally predict significant rates for rare decays such as $K \to \mu e$. This is a generic feature of almost all such models, and the fact that they have not been observed is a severe problem for the general approach and has made it difficult to construct realistic models. If one somehow evades the problem of rare decays one still generally expects to see significant effects in LEP and other precision observables, including new 4-fermi operators, decrease of the $Z \to b\bar{b}$ partial width, and modifications to ρ_0 and to the parameters S, T, and U. The fact that these have not been seen constitutes an additional serious difficulty for most such models. In the future one would also expect to see new particles and anomalous interactions among gauge bosons.

3   Recent Data

Recent results from Z-pole experiments are shown in Table 1. These include the preliminary results from the 1993 LEP energy scan, slightly updated from the values presented at the Moriond meeting. These are averages from the ALEPH, DELPHI, L3, and OPAL detectors, including a proper treatment of common systematic uncertainties [1]. In addition, the result

Quantity	Value	Standard Model
M_Z (GeV)	91.1895 ± 0.0044	input
Γ_Z (GeV)	2.4969 ± 0.0038	$2.496 \pm 0.001 \pm 0.003 \pm [0.003]$
$R = \Gamma(\text{had})/\Gamma(\ell\bar{\ell})$	20.789 ± 0.040	$20.782 \pm 0.006 \pm 0.004 \pm [0.03]$
$\sigma_{\text{had}} = \frac{12\pi}{M_Z^2} \frac{\Gamma(e\bar{e})\Gamma(\text{had})}{\Gamma_Z^2}$ (nb)	41.51 ± 0.12	$41.45 \pm 0.01 \pm 0.01 \pm [0.03]$
$R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$	0.2208 ± 0.0024	$0.2155 \pm 0 \pm 0.0004$
$R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$	0.170 ± 0.014	$0.171 \pm 0 \pm 0$
$A_{FB}^{0\ell} = \frac{3}{4}\left(A_\ell^0\right)^2$	0.0170 ± 0.0016	$0.0152 \pm 0.0005 \pm 0.0007$
$A_\tau^0\left(P_\tau\right)$	0.150 ± 0.010	$0.142 \pm 0.003 \pm 0.003$
$A_e^0\left(P_\tau\right)$	0.120 ± 0.012	$0.142 \pm 0.003 \pm 0.003$
$A_{FB}^{0b} = \frac{3}{4}A_e^0 A_b^0$	0.0960 ± 0.0043	$0.0998 \pm 0.002 \pm 0.002$
$A_{FB}^{0c} = \frac{3}{4}A_e^0 A_c^0$	0.070 ± 0.011	$0.071 \pm 0.001 \pm 0.002$
$\bar{s}_\ell^2\left(A_{FB}^Q\right)$	0.2320 ± 0.0016	$0.2321 \pm 0.0003 \pm 0.0004$
$A_e^0\left(A_{LR}^0\right)$ (SLD)	0.1637 ± 0.0075 $(92+93)$ $(0.1656 \pm 0.0076$ $(93))$	$0.142 \pm 0.003 \pm 0.003$
N_ν	2.985 ± 0.023	3

Table 1: Z-pole observables from LEP and SLD compared to their standard model expectations. The standard model prediction is based on M_Z and uses the global best fit values for m_t and α_s, with M_H in the range $60 - 1000$ GeV.

from the SLD experiment at SLAC [2] on the left-right asymmetry A_{LR} is shown. The first row in Table 1 gives the value of the Z mass, which is now known to remarkable precision. Also shown are the lineshape variables Γ_Z, R, and σ_{had}; the heavy quark production rates; various forward-backward asymmetries, A_{FB}; quantities derived from the τ polarization P_τ and its angular distribution; and the effective weak angle $\bar{s}_\ell^2$ obtained from the jet charge asymmetry. N_ν is the number of effective active neutrino flavors with masses light enough to be produced in Z decays. It is obtained by subtracting the widths for decays into hadrons and charged leptons from the total width Γ_Z from the lineshape. The asymmetries are expressed in terms of the quantity

$$A_f^o = \frac{2\bar{g}_{Vf}\,\bar{g}_{Af}}{\bar{g}_{Vf}^2 + \bar{g}_{Af}^2}, \tag{1}$$

where $\bar{g}_{V,Af}$ are the vector and axial vector couplings to fermion f.

From the Z mass one can predict the other observables, including electroweak loop effects. The predictions also depend on the top quark and Higgs mass, and α_s is needed for the QCD corrections to the hadronic widths. The predictions are shown in the third column of Table 1, using the value $m_t = 173 \pm 11$ GeV obtained for $M_H = 300$ GeV in a global best fit to all data. The first uncertainty is from M_Z and Δr (related to the running of α up to M_Z), while the second is from m_t and M_H, allowing the Higgs mass to vary in the range $60 - 1000$ GeV. The last uncertainty is the QCD uncertainty from the value of α_s. Here the value and uncertainty are given by $\alpha_s = 0.124 \pm 0.005$, obtained from the global fit to the lineshape.

The data is in excellent agreement with the standard model predictions except for two

Figure 1: Standard model prediction for $R_b \equiv \Gamma(b\bar{b})/\Gamma(\text{had})$ as a function of m_t, compared with the LEP experimental value. Also shown are the D0 lower bound of 131 GeV and the CDF range 174 ± 16 GeV.

observables. The first is

$$R_b = \frac{\Gamma(b\bar{b})}{\Gamma(\text{had})} = 0.2208 \pm 0.0024. \tag{2}$$

This is some 2.2σ higher than the standard model expectation 0.2155 ± 0.0004. Because of special vertex corrections, the $b\bar{b}$ width actually decreases with m_t, as opposed to the other widths which all increase. This can be seen in Figure 1.

It is apparent that R_b favors a small value of m_t. By itself R_b is insensitive to M_H. However, when combined with other observables, for which m_t and M_H are strongly correlated, the effect is to favor a smaller Higgs mass. Another possibility, if the effect is more than a statistical fluctuation, is that it may be due to some sort of new physics. Many types of new physics will couple preferentially to the third generation, so this is a serious possibility. However, the experimenters have not completed an analysis of the effects of Z decaying into light quarks, one of which radiates a gluon which then turns into a $b\bar{b}$. Until this is separated from the data any conclusions must be preliminary.

The other discrepancy is the value of the left-right asymmetry

$$A_{LR}^0 = A_e^0 = \frac{2\bar{g}_{Ve}\bar{g}_{Ae}}{\bar{g}_{Ve}^2 + \bar{g}_{Ae}^2} = 0.164 \pm 0.008 \tag{3}$$

obtained by the SLD collaboration. This is some 2.5σ higher than the standard model expectation of 0.142 ± 0.004. This result by itself favors a large value of the top quark mass, around 240 GeV. This certainly is not in good agreement with other observables. One possibility is that it is pointing to new physics. Possibilities here would include $S < 0$, where S is a parameter describing certain types of heavy new physics (see Section 6.5). In addition, there are possible tree-level physics such as heavy Z' bosons or mixing with heavy exotic doublet leptons, E_R', which could significantly affect the asymmetry. However, new physics probably cannot explain all of the discrepancy with the other observables, because some of the LEP observables measure precisely the same combination of couplings as does

A_{LR}[2]. In particular, the LEP collaborations measured the forward-backward asymmetry for $e^+e^- \rightarrow Z \rightarrow e^+e^-$, which yields $A_{FB}^{0e} = \frac{3}{4}A_e^{0\ 2} = 0.0158 \pm 0.0035$, implying $A_e^0 = 0.145 \pm 0.016$. Furthermore, the angular distribution of the τ polarization yields $A_e^0 = 0.120 \pm 0.012$. Combining these results,

$$A_e^0 \mid_{\text{LEP}} = 0.129 \pm 0.010 . \tag{4}$$

Thus, there is a direct experimental conflict between the LEP and SLD values of A_e^0 at the 2.7σ level. One can also consider the comparison of LEP with SLD if one assumes lepton family universality. In that case, from the forward-backward asymmetries into e, μ, and τ (A_{FB}^{0e}, $A_{FB}^{0\mu}$, $A_{FB}^{0\tau}$) one can determine the leptonic asymmetry $A_{FB}^{0\ell} = 0.0170 \pm 0.0016$, which implies $A_e^0 = 0.1505 \pm 0.0071$. Combining this with $A_e^0(P_\tau)$ and with $A_\tau^0(P_\tau) = 0.150 \pm 0.010$ (which is roughly the τ polarization averaged over angles) one obtains finally

$$A_\ell^0 \mid_{\text{LEP}} = 0.145 \pm 0.005 , \tag{5}$$

which is again 2.0σ below the SLD result.

We therefore almost certainly have an experimental conflict. It may well be due to a large statistical fluctuation. If the central value of the SLD is correct it would also call for new physics when compared with all of the other observables.

A word is in order concerning the experiments. The quantity A_{LR} measured by SLD is perhaps the cleanest single observable: it is obtained as a ratio from which most radiative corrections and systematic uncertainties cancel. It does, however, require an absolute knowledge of the beam polarization. It also has a strong sensitivity to A_e^0, even for relatively small statistics, and leads to the most precise single determination of A_e^0. The LEP results, on the other hand, are based on the averaging of a number of observations from the four groups and from various observables. None are individually as precise as the SLD result. On the other hand, the LEP experiments have done an outstanding job, and the fact that there are so many observables makes it hard to imagine that any systematic problem in one or a few of them could significantly affect the overall result.

It will take more time and more statistics to see whether this is just a fluctuation or whether there is a true discrepancy. In the meantime it leaves me with the problem of how to use the results in global fits. I will take the view that any discrepancies are statistical fluctuations, in which case the prescription is to simply combine the data. An alternative would be to multiply the error in the weighted average of A_e^0 from LEP and SLD by a scale factor $S \sim 3.1$, where S is the square root of the χ^2/df, to represent the discrepancy. The latter is the approach favored by the Particle Data Group. I will not follow it here, but will indicate its effects.

There are many other precision observables. Some recent ones are shown in Table 2. These include the D0 limit [3] $m_t > 131$ GeV and the value $m_t = 174 \pm 16$ GeV suggested by the CDF candidate events [4]. There are new observations of the W mass [5] from

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[2]The relation makes use only of the assumption that the LEP and SLD observables are dominated by the Z-pole. The one (unlikely) loophole is the possibility of an important contribution from other sources, such as new 4-fermi operators. These are mainly significant slightly away from the pole (at the pole they are out of phase with the Z amplitude and do not interfere).

Quantity	Value	Standard Model
M_W (GeV)	80.17 ± 0.18	$80.31 \pm 0.02 \pm 0.08$
$M_W/M_Z(UA2)$	0.8813 ± 0.0041	$0.8807 \pm 0.0002 \pm 0.0008$
$Q_W(C_S)$	$-71.04 \pm 1.58 \pm [0.88]$	$-72.90 \pm 0.07 \pm 0.05$
$g_A^{\nu e}$ (CHARM II)	-0.503 ± 0.018	$-0.506 \pm 0 \pm 0.001$
$g_V^{\nu e}$ (CHARM II)	-0.025 ± 0.019	$-0.038 \pm 0.001 \pm 0.001$
$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}$	0.2218 ± 0.0059 [CCFR] 0.2260 ± 0.0048 [All]	$0.2243 \pm 0.0003 \pm 0.0015$
M_H (GeV)	≥ 60 LEP	$< \begin{cases} 0(600), & \text{theory} \\ 0(800), & \text{indirect} \end{cases}$
m_t	> 131 D0 174 ± 16 CDF	$173 \pm 11^{+17}_{-19}$ [indirect]
$\alpha_s(M_Z)$	0.123 ± 0.006 LEP event shapes 0.116 ± 0.005 event shapes + low energy	$0.124 \pm 0.005 \pm 0.002$ [Z lineshape]

Table 2: Recent observables from the W mass and other non-Z-pole observations compared with the standard model expectations. Direct values and limits on M_H, m_t, and α_s are also shown.

both D0, which has presented a preliminary new value 79.86 ± 0.40 GeV, and from CDF, which finds 80.38 ± 0.23 GeV. Combining these and earlier data one obtains the results shown. Other observables include M_W/M_Z from UA2[3] [6], recent results on neutrino electron scattering from CHARM II [7], and new measurements of $s_W^2 \equiv 1 - M_W^2/M_Z^2$ from the CCFR collaboration at Fermilab [8]. This on-shell definition of the weak angle is determined from deep inelastic neutrino scattering with small sensitivity to the top quark mass. The result combined with earlier experiments [9] is also shown. All of these quantities are in excellent agreement with the standard model predictions.

In the global fits to be described, all of the earlier low energy observables not listed in the table are fully incorporated.

4   Radiative Corrections

In the electroweak theory one defines the weak angle by

$$\sin^2 \theta_W \equiv \frac{g'^2}{g^2 + g'^2} \longrightarrow \sin^2 \hat{\theta}_W(M_Z) \quad (\overline{MS}) \tag{6}$$

where g' and g are respectively the gauge couplings of the U_1 and SU_2 gauge groups. Although initially defined in terms of the gauge couplings, after spontaneous symmetry break-

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[3]I could, of course, multiply M_W/M_Z by the LEP M_Z and include the result in the M_W average. (In fact, such a procedure was carried out in the D0 analysis.) I do not do so because, in principle, it would introduce a correlation between M_Z and M_W. In practice, the effect is negligible because of the tiny uncertainty in M_Z.

ing one can relate the weak angle to the W and Z masses by

$$M_W^2 = \frac{A^2}{\sin^2 \theta_W} \longrightarrow \frac{A^2}{\sin^2 \hat{\theta}_W (1 - \Delta \hat{r}_W)} \tag{7}$$

and

$$M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W} \longrightarrow \frac{M_W^2}{\hat{\rho} \cos^2 \hat{\theta}_W (1 - \Delta \hat{r}_W)} \tag{8}$$

where

$$A^2 \equiv \frac{\pi \alpha}{\sqrt{2} G_F} = (37.2802 \text{ GeV})^2 . \tag{9}$$

The first form of equations (6)–(8) are valid at tree level. However, the data is sufficiently precise that one must include full one loop radiative corrections, which means that one must replace the quantities by the expressions shown in the last part of equations. There are a number of possible ways of defining the renormalized weak angle. Here I am using the quantity $\sin^2 \hat{\theta}_W (M_Z)$, which is renormalized according to modified minimal subtraction, $\overline{MS}$ [10]. This basically means that one removes the $\frac{1}{n-4}$ poles and some associated constants from the gauge couplings. In equation (7) the quantity $\Delta \hat{r}_W$ contains the finite radiative corrections which relate the W and Z masses, muon decay, and QED. The dominant contribution is given by the running of the fine structure constant α from low energies, where it is defined in QED, up to the Z-pole, which is the scale relevant for electroweak interactions,

$$\frac{1}{1 - \Delta \hat{r}_W} \simeq \frac{\alpha(M_Z)}{\alpha} \sim \frac{1/128}{1/137}. \tag{10}$$

There is only a weak dependence on the top quark mass in this scheme, leading to a value

$$\Delta \hat{r}_W \sim 0.07 \tag{11}$$

dominated by the running of α. There is a theoretical uncertainty from the contribution of light hadrons to the photon self-energy diagrams. This leads to a theoretical uncertainty of ± 0.0009. This turns out to be the dominant theoretical uncertainty in the precision electroweak tests and, in particular, in the expressions relating the Z mass to other observables. A similar effect leads to a significant theoretical uncertainty in $g_\mu - 2$, which will dominate the experimental uncertainties in the new Brookhaven experiment unless associated measurements are made of the cross-section for $e^+ e^- \rightarrow$ hadrons at low energies.

Because m_t is so much heavier than the bottom quark mass there is large SU_2 breaking generated by loop diagrams involving the top and bottom quarks, in particular from the W and Z self-energy diagrams. There is little shift in the W mass, because that effect is already absorbed into the observed value of the Fermi constant, so $\Delta \hat{r}_W$ has no large m_t dependence. However, the Z mass prediction is shifted down. In particular, the quantity $\hat{\rho}$ in equation (8) depends quadratically on m_t. It is given by [11] $\hat{\rho} \sim 1 + \rho_t$, where $\rho_t = \frac{3 G_F m_t^2}{8 \sqrt{2} \pi^2} \sim 0.0031 (m_t / 100 \text{ GeV})^2$. (There are additional contributions from bosonic loops.) For m_t in the range $100 - 200$ GeV the effect on $\hat{\rho}$ can be quite significant. ρ_t propagates to other observables and generates most of the major m_t dependence. (The one exception is the vertex correction to $Z \rightarrow b\bar{b}$ decay.)

Figure 2: Values of $\sin^2 \hat{\theta}_W(M_Z)$ as a function of m_t from various observables.

From the precise value $M_Z = 91.1895 \pm 0.0044$ GeV from LEP one has

$$\sin^2 \hat{\theta}_W(M_Z) = 0.2318 \pm 0.0005. \tag{12}$$

The uncertainty is an order of magnitude smaller than one had prior to the Z-pole experiments at LEP. The uncertainty from the experimental error in the Z mass is negligible, of order 0.00003. The theoretical uncertainty 0.0003 coming from $\Delta \hat{r}_W$ is much larger. The largest uncertainty, however, is from m_t and M_H, ~ 0.0004. Here I have used the range of m_t from the global best fit, and 60 GeV $< M_H < 1000$ GeV. If one knew m_t one would have a more precise value of the weak angle. The sensitivity is displayed in Figure 2. Clearly, one cannot determine the weak angle from M_Z alone because of the m_t dependence. One must have either other indirect observables with a different dependence on m_t or a direct measurement. Before discussing other possibilities, I will digress somewhat on the radiative corrections [10].

The radiative corrections fall into three categories. First, there are the reduced QED corrections, which involve the emission of real photons and the exchange of virtual photons but do not include vacuum polarization diagrams. These constitute a gauge invariant set, but depend on the details of the experimental acceptances and cuts. They generally are removed from the data by the experimenters. The second class has already been described. It is the electromagnetic vacuum polarization diagrams, which lead to the running from $\alpha^{-1} \sim 137$ at low energies to $\alpha(M_Z)^{-1} \sim 128$ at the Z-pole. As we have seen this leads to a significant uncertainty $\Delta \hat{r}_W \sim \Delta\alpha(M_Z)/\alpha \sim 0.0009$, which can lead to a shift of approximately 3 GeV in the predicted value of m_t.

The electroweak corrections are now quite important. One must include full 1-loop corrections as well as dominant 2-loop effects. The electroweak corrections include and are dominated by the gauge self-energy diagrams for the W, Z, and γZ mixing. In addition, there are box diagrams and vertex corrections, which are smaller but which have to be included. Recently there has been some progress on the dominant 2-loop effects. In particular, the dominant terms of order $\alpha^2 m_t^4$ are included. The net effect is to replace [12]

$$\hat{\rho} \to 1 + \rho_t \left[1 + \rho_t R \left(\frac{M_H}{m_t} \right) \right], \tag{13}$$

where

$$\rho_t = \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2},\tag{14}$$

and R, which comes from 2-loop diagrams, is strongly dependent on M_H, with $R(0) = 19 - 2\pi^2$. There are additional smaller contributions which must be included in the numerical analysis.

There are also significant mixed QCD-electroweak diagrams, such as those obtained by the exchange of the gluon across the quarks in a self-energy diagram. The dominant contribution involves top quark loops and is of order $\alpha\alpha_s m_t^2$. This leads to the replacement

$$\hat{\rho} \to 1 + \rho_t \left[1 - 2\alpha_s(m_t)\frac{\pi^2 + 3}{9\pi} \right] \sim 1 + 0.9\rho_t,\tag{15}$$

which raises the predicted value of m_t by approximately 5%. Recently there have been discussions and estimates of $t\bar{t}$ threshold corrections, which are $O(\alpha\alpha_s^2 m_t^2)$. These have been estimated using both perturbative [13] methods and by dispersion relations [14]. One estimate [13] is that the effect is mainly to shift the scale at which α_s should be evaluated for the t quark loop, namely $\alpha_s(m_t) \to \alpha_s(0.15 m_t)$. This is in good numerical agreement with the dispersion relation estimate. The threshold estimates have not been included here, but would raise the predicted values of m_t by +3 GeV.

4.1 The Great Confusion: $\sin^2\theta_W$

There are a number of different definitions of the renormalized weak angle used in the literature, leading to considerable confusion. Each of the definitions has its advantages and disadvantages. At tree-level there are several equivalent expressions, namely

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = 1 - \frac{M_W^2}{M_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F M_W^2}.\tag{16}$$

The first definition is based on the coupling constants; the last two take meaning only after spontaneous symmetry breaking has occurred, and therefore mix in parts of the theory in addition to the gauge vertices. At higher order one must define a renormalized angle. One can use the different expressions in equation (16) as starting points, and the resulting definitions differ by finite terms of order α, which also depend on m_t and M_H. This has lead to considerable confusion (and sometimes heat).

Two common definitions are based on the spontaneous symmetry breaking (SSB) of the theory, namely on the gauge boson masses. The most famous is the on-shell definition [10]

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} = 0.2242 \pm 0.0012.\tag{17}$$

This is very simple conceptually. However, the W mass is not determined as precisely as M_Z, so s_W^2 must actually be extracted from other data and not from the defining relation (17).

This leads to a strong dependence on m_t, which accounts for almost all of the uncertainty in s_W^2. (The value for s_W^2 and the other definitions is from a global fit to all data.)

The Z-mass definition [15],

$$s_{M_Z}^2 \left(1 - s_{M_Z}^2\right) = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2} = 0.2312 \pm 0.0003, \tag{18}$$

is obtained by simply removing the m_t dependence from the expression for the Z mass. This is the most precise – the uncertainty is mainly from $\alpha(M_Z)$. The use of $s_{M_Z}^2$ is essentially equivalent to using the Z mass as a renormalized parameter, introducing the weak angle as a useful derived quantity. This scheme is simple and precise, and by definition there is no m_t dependence in the relation betwen M_Z and $s_{M_Z}^2$. However, the m_t dependence and uncertainties enter as soon as one tries to predict other quantities in terms of it.

Both of the definitions based on spontaneous symmetry breaking tend to be awkward in the presence of new physics, which might shift the values of the gauge boson masses. There are other definitions based on the gauge coupling constants. These are especially useful for applications to grand unification, and they tend to be less sensitive to the presence of new physics. One is the modified minimal subtraction or ($\overline{MS}$) definition [10]

$$\hat{s}_Z^2 = \frac{\hat{g}'^2(M_Z)}{\hat{g}'^2(M_Z) + \hat{g}^2(M_Z)} = 0.2317 \pm 0.0004, \tag{19}$$

defined by removing the poles and associated constants from the gauge couplings. As we have seen, the uncertainty is mainly from $\alpha(M_Z)$ and m_t. There are variant definitions of $\hat{s}_Z^2$, depending on the treatment of $\alpha \ln(m_t/M_Z)$ terms. One cannot decouple all such terms because $m_t \gg m_b$ breaks SU_2. The version used here [16] decouples them from $\gamma - Z$ mixing, essentially eliminating any m_t dependence from the Z-pole asymmetry formulas.

Finally, the experimental groups at LEP and SLC have made extensive use of

$$\begin{aligned} \bar{g}_{Af} &= \sqrt{\rho_f}\, t_{3f} \\ \bar{g}_{Vf} &= \sqrt{\rho_f}\left[t_{3f} - 2\bar{s}_f^2 q_f\right]. \end{aligned} \tag{20}$$

These are the effective axial and vector couplings of the Z to fermion f. In equation (20) $t_{3f} = \pm\frac{1}{2}$ is the weak isospin of fermion f and q_f is its electric charge. The electroweak self-energy and vertex corrections are absorbed into the coefficient ρ_f and the effective weak angle $\bar{s}_f^2$. The $\bar{g}_{V,Af}$ are obtained from the data after removing all photonic contributions. In principle there are also electroweak box contributions. However, these are very small and are typically ignored or removed from the data.

The effective weak angle differs for different fermions. $\bar{s}_f^2$ is related to the $\overline{MS}$ angle, for example, by

$$\bar{s}_f^2 = \kappa_f \hat{s}_Z^2 \tag{21}$$

where κ_f is a form factor. The best measured is for the charged leptons, for which

$$\bar{s}_\ell^2 \sim \hat{s}_Z^2 + 0.0002 = 0.2319 \pm 0.0004 \tag{22}$$

On-shell : $s_W^2 = 1 - \frac{M_W^2}{M_Z^2} = 0.2242\,(12)$
+ most familiar
+ simple conceptually
− large m_t dependence from Z-pole observables
− depends on SSB mechanism − awkward for new physics

Z-mass : $s_{M_Z}^2 = 0.2312(3)$
+ most precise (no m_t dependence)
+ simple conceptually
− m_t reenters when predicting other observables
− depends on SSB mechanism − awkward for new physics

$\overline{MS}$: $\hat{s}_Z^2 = 0.2317\,(4)$
+ based on coupling constants
+ convenient for GUTs
+ usually insensitive to new physics
+ Z asymmetries $\sim$ independent of m_t
− theorists definition; not simple conceptually
− usually determined by global fit
− some sensitivity to m_t
− variant forms (m_t cannot be decoupled in all processes ($\hat{s}_{ND}^2$ larger by $0.0001 - 0.0002$)

effective : $\bar{s}_\ell^2 = 0.2319 \pm 0.0004$
+ simple
+ Z asymmetry independent of m_t
+ Z widths: m_t in ρ_f only
− phenomenological; exact definition in computer code
− different for each f
− hard to relate to non Z-pole observables

Table 3: Advantages and disadvantages of several definitions of the weak angle.

where there is an additional theoretical uncertainty of ± 0.0001 from the precise definition of the angles and higher order effects. These effective angles are very simple for the discussion of the Z-pole data, but they are difficult to relate to other types of observables. All of these definitions have advantages and disadvantages, some of which are listed in Table 3.

4.2 Other Z-Pole Observables

The other Z-pole observables can also be computed. For example, the partial width for Z to decay into fermions $f\bar{f}$ is given approximately by

$$\Gamma(f\bar{f}) \simeq C_F \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left[|\bar{g}_{Af}|^2 + |\bar{g}_{Vf}|^2 \right]. \tag{23}$$

For the heavier quarks and leptons kinematic mass corrections must be applied. Effective couplings are proportional to $\sqrt{\bar{\rho}}$ so that each partial width increases quadratically with m_t.

This comes from the replacement

$$\frac{M_Z g^2}{8 \cos^2 \hat{\theta}_W} \rightarrow \hat{\rho}\frac{G_F}{\sqrt{2}}M_Z^3,\tag{24}$$

which incorporates many of the low energy corrections. In equation (23) there is an additional coefficient

$$C_f = \begin{cases} 1 + \frac{3\alpha}{4\pi}q_f^2 , & \text{leptons} \\ 3\left(1 + \frac{3\alpha}{4\pi}q_f^2\right)\left(1 + \frac{\alpha_s}{\pi} + 1.409\left(\frac{\alpha_s}{\pi}\right)^2 - 12.77\left(\frac{\alpha_s}{\pi}\right)^3\right) & \text{quarks} \end{cases},\tag{25}$$

which includes QED and QCD corrections. In particular, the α_s dependence of the hadronic widths leads to a determination of $\alpha_s = 0.124 \pm 0.005$ just from the lineshape. For fixed M_Z most of the m_t dependence is in the $\hat{\rho}$ factor. One major exception[4] is that $\Gamma(b\bar{b})$ decreases with m_t due to special m_t-dependent vertex corrections [17], [18]. These are included in the ρ_b and κ_b factors, but to an excellent numerical approximation $\Gamma(b\bar{b})$ can be written as [18],

$$\Gamma(b\bar{b}) \rightarrow \Gamma^0(b\bar{b})\left(1 + \delta_{bb}^{SM}\right) \sim \Gamma^0(b\bar{b})\left[1 - 10^{-2}\left(\frac{m_t^2}{2M_Z^2} - \frac{1}{5}\right)\right],\tag{26}$$

where $\Gamma^0(b\bar{b})$ is the standard model expression without the corrections. This special dependence is useful for separating the m_t and Higgs effects.

In addition there are various asymmetries observed at LEP and SLD. In particular, the forward-backward asymmetry for $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ is given, after removing photonic effects and boxes, by

$$A_{FB}^{0f} \simeq \frac{3}{4}A_e^0 A_f^0,\tag{27}$$

where A_f^0 is defined in (1). Other asymmetries include the polarization of produced τ's. From the angular distribution of the τ polarization one can obtain A_τ^0 and A_e^0, with A_τ^0 coming mainly from the average polorization and A_e^0 mainly from its forward-backward asymmetry. The SLD collaboration has polarized electrons; from the left-right asymmetry as the polarization is reversed one can also determine A_e^0, namely $A_{LR}^0 = A_e^0$.

All of these asymmetries are independent of m_t when expressed in terms of the effective angles $\bar{s}_f^2$ and almost independent of m_t when expressed in terms of the $\overline{MS}$ angle $\hat{s}_Z^2$. One can therefore determine $\bar{s}_\ell^2$ or $\hat{s}_Z^2$ from the data without theoretical uncertainties from m_t. On the other hand, in the on-shell or Z-mass schemes the formulas involve quadratic m_t dependence.

5   Results: m_t, M_H, α_s, $\sin^2\theta_W$

There are now sufficiently many observables that one can precisely determine $\hat{s}_Z^2$, m_t, and $\alpha_s(M_Z)$ simultaneously. For example, $\hat{s}_Z^2$ can be determined from the asymmetries, m_t from the W and Z masses, and $\alpha_s(M_Z)$ from the hadronic Z-widths. In practice all of these

Set	$\hat{s}_Z^2$	$\alpha_s(M_Z)$	m_t (GeV)	$\Delta\chi_H^2$
All indirect	0.2317(3)(2)	0.124(5)(2)	$173\pm11^{+17}_{-19}$	3.3
Indirect + CDF (174 ± 16)	0.2317(3)(3)	0.124(5)(2)	$174 \pm 9 \pm 12$	3.0
LEP + low energy	0.2321(4)(2)	0.126(5)(2)	$165^{+11}_{-12}{}^{+17}_{-19}$	1.6
All indirect $(S = 3.1)$	0.2319(4)(2)	0.125(5)(2)	$169^{+11}_{-12}{}^{+17}_{-19}$	2.5
Z-pole	0.2316(4)(2)	0.124(5)(2)	$178^{+11}_{-12}{}^{+17}_{-19}$	3.1
LEP	0.2320(4)(2)	0.126(5)(2)	$168^{+12}_{-13}{}^{+17}_{-19}$	1.5
SLD + M_Z	0.2291(10)(0)	—	$244^{+23}_{-25}{}^{+19}_{-22}$	

Table 4: Results for the electroweak parameters in the standard model from various sets of data. The central values assume $M_H = 300$ GeV, while the second errors are for $M_H \to 1000(+)$ and $60(-)$. The last column is the increase in the overall χ^2 of the fit as M_H increases from 60 to 1000. m_t would increase by some 3 GeV in the fits to the indirect data if one included the estimates of the $\alpha\,\alpha_s^2\,m_t^2$ threshold corrections.

quantities are determined from a simultaneous fit. The results of fits to various sets of data are shown in Table 4. The first row of the table includes the global fit to all indirect data. The predicted value of m_t is in remarkable agreement with the value 174 ± 16 GeV suggested by the CDF candidate events [4]. The second row includes the direct (CDF) value for m_t as a separate constraint. The other fits show the sensitivity to the various data sets. The third row includes the LEP results and the low energy data but not SLD. Comparing with the first row one sees that the predicted m_t is pulled up significantly (by ~ 10 GeV) by the SLD result. The next row combines the LEP and SLD measurements of A_e using the scale factor of 3.1. The last rows are the result of the Z-pole, LEP, and SLD observables by themselves.

Using the results of the 1993 LEP energy scan we can now extract the strong coupling constant α_s at the Z-pole with a small experimental error,

$$\alpha_s(M_Z) = 0.124 \pm 0.005 \pm 0.002 \quad \text{(lineshape)}, \tag{28}$$

where the second uncertainty is from M_H. α_s is almost uncorrelated with the other parameters. It is determined mainly from the ratio $R \equiv \Gamma(\text{had})/\Gamma(\ell\bar{\ell})$, which is insensitive to m_t (except in the $b\bar{b}$ vertex), and also from Γ_Z. This determination is very clean theoretically, at least within the standard model. It is the Z-pole version of the long held view that the ratio of hadronic to leptonic rates in e^+e^- would be a "gold plated" extraction of α_s and test of QCD. Using a recent estimate [19] of the $(\alpha_s/\pi)^4$ corrections to C_F, i.e. $-90(\alpha_s/\pi)^4$, one can estimate that higher-order terms lead to an additional uncertainty $\sim \pm0.001$ in the $\alpha_s(M_Z)$ value in (28). It should be cautioned, however, that the lineshape value is rather sensitive to the presence of some types of new physics.

The lineshape value of α_s is an excellent agreement with the independent value $\alpha_s(M_Z) = 0.123 \pm 0.005$ extracted from jet event shapes at LEP using resummed QCD [20]. It is also in excellent agreement with the prediction

$$\alpha_s(M_Z) \sim 0.127 \pm 0.008, \quad \text{SUSY} - \text{GUT} \tag{29}$$

---

[4]There is also an indirect m_t dependence in $\bar{s}_f^2$ if one regards M_Z as fixed.

Source	$\alpha_s(M_Z)$
R_τ	0.122 ± 0.005
Deep inelastic	0.112 ± 0.005
Υ, J/Ψ	0.113 ± 0.006
Charmonium spectrum (lattice)	0.110 ± 0.006
LEP, lineshape	$0.124 \pm 0.005 \pm 0.002$
LEP, event topologies	0.123 ± 0.005

Table 5: Values of α_s at the Z-pole extracted from various methods.

of supersymmetric grand unification. As can be seen in Table 5, however, it is somewhat larger than some of the low energy determinations of α_s (which are then extrapolated theoretically to the Z-pole), in particular those from deep inelastic scattering and the lattice calculation of the charmonium spectrum[5]. This slight discrepancy has led some authors to suggest that there might be a light gluino which would modify the running of α_s. I think, however, that it is premature to draw such a strong conclusion. It should be noted that there is an independent low energy LEP determination from the ratio R_τ of hadronic to leptonic τ decays, which gives a larger value.

5.1   The Higgs Mass

The new data also constrain the Higgs boson mass. This enters $\hat{\rho}$ logarithmically and is strongly correlated with the quadratic m_t dependence in everything but the $Z \rightarrow b\bar{b}$ vertex correction. The χ^2 distribution as a function of the Higgs mass is shown in Figure 3; the minimum occurs at the lower limit, 60 GeV, allowed by direct searches at LEP, or at ~ 120 GeV when the CDF m_t value is included. These low values are consistent with the minimal supersymmetric extension of the standard model, which generally predicts a relatively light standard model-like Higgs scalar. However, the constraint is very weak statistically. From the χ^2 distribution one obtains the weak upper limits

$$\text{indirect}: \quad M_H < 780(1160)\text{GeV} \tag{30}$$

at 90 (95)% CL from the indirect precision data, and

$$\text{indirect} + \text{CDF} \; : \quad M_H < 740(1040)\text{GeV} \tag{31}$$

including the CDF direct constraint from m_t. (These results include the direct limit $M_H > 60$ GeV.) Clearly, no definitive conclusion can be drawn. An additional strong caveat is in order: the preference for small M_H is driven almost entirely by $\Gamma(b\bar{b})$, which is significantly above the standard model prediction even for $M_H = 60$ GeV. If that is due to a large statistical fluctuation or to some new physics then the constraint on M_H would essentially disappear. Finally, the statistical significance of the result would decrease even more if the SLC result were omitted.

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[5]The lattice value 0.110 ± 0.006 [21] has increased somewhat from the published value of 0.105 ± 0.004 [22], reducing the discrepancy.

Figure 3: χ^2 distributions of the overall fits as a function of M_H.

The weak M_H dependence does not imply that the data is insensitive to the spontaneous symmetry breaking mechanisms. Alternative schemes generally yield large effects on the precision observables, as will be described below.

5.2 Have Electroweak Corrections Been Seen?

The data can also be interpreted in terms of whether one has actually observed the electroweak (as opposed to the simple running α) corrections. Novikov *et al.* [15] have noted that there is a large cancellation between the fermionic and bosonic contributions to the W and Z self-energies, and that until the most recent data the data could actually be fit by a properly interpreted Born theory. However, the data is now sufficiently good that even given the cancellations these electroweak loops are needed at the 2σ level. Gambino and Sirlin [23] and Schildknecht [24] have interpreted the data in somewhat different way. They have argued that the fermionic loops, both in the running of α and the t, b loops, are unambiguous theoretically, and certainly should be there if the theory is to make any sense. However, the bosonic loops, which involve triple-gauge vertices, gauge-Higgs vertices, etc., have never been independently tested in other processes. They have shown that the data are inconsistent if one simply ignores bosonic loops (which are a gauge-invariant subset of diagrams), thus providing convincing though indirect evidence for their existence.

6 New Physics

6.1 Supersymmetry and Precision Experiments

Let us now consider how the predictions for the precision observables are modified in the presence of supersymmetry. There are basically three implications for the precision results. The first, and most important, is in the Higgs sector. In the standard model the Higgs mass is arbitrary. It is controlled by an arbitrary quartic Higgs coupling, so that M_H could be as small as 60 GeV (the experimental limit) or as heavy as a TeV. The upper bound

is not rigorous: larger values of M_H would correspond to such large quartic couplings that perturbation theory would break down. This cannot be excluded, but would lead to a theory that is qualitatively different from the (perturbative) standard model. In particular, there are fairly convincing triviality arguments, related to the running of the quartic coupling, which exclude a Higgs which acts like a distinct elementary particle for M_H above $O(600 \text{ GeV})$ [25].

However, in supersymmetric extensions of the standard model the quartic coupling is no longer a free parameter. It is given by the squares of gauge couplings, with the result that all supersymmetric models have at least one Higgs scalar that is relatively light, typically with a mass similar to the Z mass. In the minimal supersymmetric standard model (MSSM) one has $M_H < 150 \text{ GeV}$[6], which generally acts just like the standard model Higgs[7] except that it is necessarily light.

In the standard model there is a large $m_t - M_H$ correlation, and one has the prediction

$$m_t \sim 173 \pm 11 + 13 \ln \left(\frac{M_H}{300 \text{GeV}} \right). \tag{32}$$

We have seen that for $60 < M_H < 1000$ GeV this corresponds to

$$m_t = 173 \pm 11^{+17}_{-19} \text{ (SM)}. \tag{33}$$

However, in MSSM one has the smaller range $60 < M_H < 150$ GeV, leading to the lower prediction

$$m_t = 159^{+11}_{-12} \pm 5 \text{ (MSSM)}. \tag{34}$$

This is on the low side of the CDF range, $(174 \pm 16 \text{ GeV})$, but not excluded.

There can be additional effects on the radiative corrections due to sparticles and the second Higgs doublet that must be present in the MSSM. However, for most of the allowed parameter space one has $M_{\text{new}} \gg M_Z$, and the effects are negligible by the decoupling theorem. For example, a large $\tilde{t} - \tilde{b}$ splitting would contribute to the ρ_0 (SU_2-breaking) parameter (to be discussed below), leading to a smaller prediction for m_t, but these effects are negligible for $m_{\tilde{q}} \gg M_Z$. Similarly, there would be new contributions to the $Z \to b\bar{b}$ vertex for $m_{\chi^\pm}$, $m_{\tilde{t}}$, or $M_H^\pm \sim M_Z$.

There are only small windows of allowed parameter space for which the new particles contribute significantly to the radiative corrections. Except for these, the only implications of supersymmetry from the precision observables are: (a) there is a light standard model-like Higgs, which in turn favors a smaller value of m_t. Of course, if a light Higgs were observed it would be consistent with supersymmetry but would not by itself establish it. That would require the direct discovery of the superpartners, probably at the LHC. (b) Another important implication of supersymmetry, at least in the minimal model, is the *absence* of other deviations from the standard model predictions. (c) In supersymmetric grand unification one expects the gauge coupling constants to unify when extrapolated from

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[6]At tree-level, $M_H < M_Z$.
[7]This is true if the second Higgs doublet is much heavier than M_Z.

their low energy values [26]. This is consistent with the data in the MSSM but not in the ordinary standard model (unless other new particles are added). This is not actually a modification of the precision experiments, but a prediction for the observed gauge couplings. Of course, one could have supersymmetry without grand unification.

6.2 Extended Technicolor/Compositeness

In contrast, the other major class of extensions, which includes compositeness and dynamical symmetry breaking, leads to many implications at low energies. The most important are large flavor changing neutral currents (FCNC). Even if these are somehow evaded one generally expects anomalous contributions to the $Z \to b\bar{b}$ vertex, typically $\Gamma(b\bar{b}) < \Gamma^{SM}(b\bar{b})$ in the simplest extended technicolor (ETC) models [27]. Similarly, one expects $\rho_0 \neq 1$, and $S_{\text{new}} \neq 0, T_{\text{new}} \neq 0$, where ρ_0, S_{new}, and T_{new} parameterize certain types of new physics, as will be described below. Finally, in theories with composite fermions one generally expects new 4-fermi operators generated by constituent interchange, leading to effective interactions of the form

$$L = \pm \frac{4\pi}{\Lambda^2} \bar{f}_1 \Gamma f_2 \bar{f}_3 \hat{\Gamma} f_4. \tag{35}$$

Generally, the Z-pole observables are not sensitive to such operators, since they only measure the properties of the Z and its couplings[8]. However, low energy experiments are sensitive. In particular, FCNC constraints typically set limits of order $\Lambda \geq O(100 \text{ TeV})$ on the scale of the operators unless the flavor-changing effects are fine-tuned away. Even then there are significant limits from other flavor conserving observables. For example, atomic parity violation [28] is sensitive to operators such as [29]

$$L = \pm \frac{4\pi}{\Lambda^2} \bar{e}_L \gamma_\mu e_L \bar{q}_L \gamma^\mu q_L. \tag{36}$$

The existing data already sets limits $\Lambda > O(10 \text{ TeV})$. Future experiments should be sensitive to ~ 40 TeV.

6.3 The $Zb\bar{b}$ Vertex

The $Zb\bar{b}$ vertex is especially interesting, both in the standard model and in the presence of new physics. In the standard model there are special vertex contributions which depend quadratically on the top quark mass, which are shown approximately in (26). $\Gamma(b\bar{b})$ actually decreases with m_t, as opposed to other widths which all increase due to the $\hat{\rho}$ parameter. The m_t and M_H dependences in $\hat{\rho}$ are strongly correlated, but the special vertex corrections to $\Gamma(b\bar{b})$ are independent of M_H, allowing a separation of m_t and M_H effects.

The vertex is also sensitive to a number of types of new physics. One can parameterize such effects by [30]

$$\Gamma(b\bar{b}) \to \Gamma^{SM}(b\bar{b})\left(1 + \delta_{bb}^{\text{new}}\right) \sim \Gamma^0(b\bar{b})\left(1 + \delta_{bb}^{\text{SM}} + \delta_{bb}^{\text{new}}\right). \tag{37}$$

---

[8]At the Z-pole the effects of new operators are out of phase with the Z amplitude and do not interfere. Interference effects can survive away from the pole, but there the Z amplitude is smaller.

If the new physics gives similar contributions to vector and axial vector vertices then the effects on A_{FB}^b are negligible. In supersymmetry one can have both positive and negative contributions [31]. In particular, light $\tilde{t} - \chi^{\pm}$ can give $\delta_{bb}^{\mathrm{SUSY}} > 0$, as is suggested by the data, while light charged Higgs particles can yield $\delta_{bb}^{\mathrm{Higgs}} < 0$. In practice, both effects are too small to be important in most allowed regions of parameter space. In extended technicolor (ETC) models there are typically new vertex contributions generated by the same ETC interactions which are needed to generate the large top quark mass. It has been argued that these are typically large and negative [27],

$$\delta_{bb}^{\mathrm{ETC}} \sim -0.056\xi^2 \left(\frac{m_t}{150\mathrm{GeV}} \right), \tag{38}$$

where ξ is a model dependent parameter of order unity. They may be smaller in models with walking technicolor, but nevertheless are expected to be negative and significant [32]. This is in contrast to the data, which suggests a positive contribution if any, implying a serious problem for many ETC models. One possible way out are models in which the ETC and electroweak groups do not commute, for which either sign is possible [33].

Another possibility is mixing between the b and exotic heavy fermions with non-canonical weak interaction quantum numbers. Many extensions of the standard model predict, for example, the existence of a heavy D_L, D_R, which are both SU_2 singlet quarks with charge $-1/3$. These can mix with the d, s, or b quarks, but one typically expects such mixing to be largest for the third generation. However, this mechanism gives a negative contribution

$$\delta_{bb}^{D_L} \sim -2.3s_L^2 \tag{39}$$

to $\delta_{bb}^{\mathrm{new}}$, where s_L is the sine of the $b_l - D_L$ mixing angle.

One can extract $\delta_{bb}^{\mathrm{new}}$ from the data, in a global fit to the standard model parameters as well as $\delta_{bb}^{\mathrm{new}}$. This yields

$$\delta_{bb}^{\mathrm{new}} = 0.031 \pm 0.014, \tag{40}$$

which is $\sim 2.2\sigma$ above zero. This value is hardly changed when one allows additional new physics, such as described by the S, T, and U parameters. Note that $\delta_{bb}^{\mathrm{new}}$ is correlated with $\alpha_s(M_Z)$: one obtains $\alpha_s(M_Z) = 0.103 \pm 0.011$, considerably smaller than the standard model value $0.124(5)(2)$. Allowing $\delta_{bb}^{\mathrm{new}} \neq 0$ has negligible effect on $\hat{s}_Z^2$ or m_t.

6.4 ρ_0: Nonstandard Higgs or Non-degenerate Heavy Multiplets

One parameterization of certain new types of physics is the parameter ρ_0, which is introduced to describe new sources of SU_2 breaking other than the ordinary Higgs doublets or the top/bottom splitting. New physics can affect ρ_0 at either the tree or loop-level

$$\rho_0 = \rho_0^{\mathrm{tree}} + \rho_0^{\mathrm{loop}}. \tag{41}$$

The tree-level contribution is given by Higgs representations larger than doublets, namely,

$$\rho_0^{\mathrm{tree}} = \frac{\sum_i \left(t_i^2 - t_{3i}^2 + t_i \right) |\langle \phi_i \rangle|^2}{\sum_i 2t_{3i}^2 |\langle \phi_i \rangle|^2}, \tag{42}$$

where t_i (t_{3i}) is the weak isospin (third component) of the neutral Higgs field ϕ_i. If one has only Higgs singlets and doublets ($t_i = 0, \frac{1}{2}$), then $\rho_0^{\text{tree}} = 1$. However, in the presence of larger representations with non-zero vacuum expectation values

$$\rho_0^{\text{tree}} \simeq 1 + 2\sum_i \left(t_i^2 - 3t_{3i}^2 + t_i\right) \frac{|\langle\phi_i\rangle|^2}{|\langle\phi_{\frac{1}{2}}\rangle|^2}. \tag{43}$$

One can also have loop-induced contributions similar to that of the top/bottom, due to non-degenerate multiplets of fermions or bosons. For new doublets

$$\rho_0^{\text{loop}} = \frac{3G_f}{8\sqrt{2}\pi^2} \sum_i \frac{C_i}{3} F(m_{1i}, m_{2i}), \tag{44}$$

where $C_i = 3(1)$ for color triplets (singlets) and

$$F(m_1, m_2) = m_1^2 + m_2^2 - \frac{4m_1^2\, m_2^2}{m_1^2 - m_2^2}\ln\frac{m_2}{m_2} \geq (m_1 - m_2)^2. \tag{45}$$

Loop contributions to ρ_0 are generally positive,[9] and if present would lead to lower values for the predicted m_t. ρ_0^{tree} can be either positive or negative depending on the quantum numbers of the Higgs field. The ρ_0 parameter is extremely important because one expects $\rho_0 \sim 1$ in most superstring theories, which generally do not have higher-dimensional Higgs representations, while typically $\rho_0 \neq 1$ from many sources in models involving compositeness.

In the presence of ρ_0 the standard model formulas for the observables are modified. One has

$$M_Z \to \frac{1}{\sqrt{\rho_0}} M_Z^{SM}, \Gamma_Z \to \rho_0 \Gamma_Z^{SM}, \mathcal{L}_{NC} \to \rho_0 \mathcal{L}_{NC}^{SM}. \tag{46}$$

It has long been known that ρ_0 is close to 1. However, until recently it has been difficult to separate ρ_0 from m_t, because in most observables one has only the combination $\rho_0\hat{\rho}$. The one exception has been the $Z \to b\bar{b}$ vertex. However, assuming that CDF has really observed the top quark directly one can use the known m_t to calculate $\hat{\rho}$ and therefore separate ρ_0. In practice one fits to m_t, ρ_0 and the other parameters, using the CDF value $m_t = 174\pm16$ GeV as an additional constraint. One can determine $\hat{s}_Z^2$, ρ_0, m_t, and α_s simultaneously, yielding

$$\begin{array}{ll} \hat{s}_Z^2 = 0.2316(3)(2) & \rho_0 = 1.0009 \pm 0.0018 \pm 0.0017 \\ \alpha_s = 0.123(6)(1) & m_t = 167 \pm 15 \pm 1 \text{ GeV}, \end{array} \tag{47}$$

where the second uncertanty is from M_H. Even in the presence of the classes of new physics parameterized by ρ_0 one still has robust predictions for the weak angle and a good determination of α_s. Most remarkably, given the CDF constraint, ρ_0 is constrained to be very close to unity, causing serious problems for compositeness models. The allowed region in ρ_0 vs $\hat{s}_Z^2$ are shown in Figure 4. This places limits $|\langle\phi_i\rangle|/|\langle\phi_{1/2}\rangle| < $ few% on non-doublet vacuum expectation values, and places constraints $\frac{C}{3}F(m_1, m_2) \leq (100 \text{ GeV})^2$ on the splittings of additional fermion or boson multiplets.

---

[9]One can have $\rho^{\text{loop}} < 0$ for Majorana fermions [34] or boson multiplets with vacuum expectation values [35].

Figure 4: Allowed regions in ρ_0 vs $\hat{s}_Z^2$ for $M_H = 60$, 300, and 1000 GeV.

6.5   Heavy Physics by Gauge Self Energies

A larger class of extensions of the standard model can be parameterized by the S, T and U parameters [36], which describe that subset of new physics which affect only the gauge boson self-energies but do not directly affect new vertices, etc. One introduces three parameters

$$
\begin{aligned}
S &= S_{\text{new}} + S_{m_t} + S_{M_H} \\
T &= T_{\text{new}} + T_{m_t} + T_{M_H} \\
U &= U_{\text{new}} + U_{m_t}.
\end{aligned}
\tag{48}
$$

S describes the breaking of the SU_{2A} axial generators and is generated, for example, by degenerate heavy chiral families of fermions. T and U describe the breaking of SU_{2V} vector generators: T is equivalent to the ρ_0 parameter and is induced by mass splitting in multiplets of fermions or bosons. U is zero in most extensions of the standard model. S, T and U were introduced to describe the contributions of new physics. However, they can also parametrize the effects of very heavy m_t and M_H (compared to M_Z). Until recently it was difficult to separate the m_t and new physics contributions. Now, however, with the CDF value of m_t it is possible to directly extract the new physics contributions.

A new multiple of degenerate chiral fermions will contribute to S_{new} by

$$
S_{\text{new}}|_{\text{degenerate}} = C_i |t_{3L}(i) - t_{3R}(i)|^2 / 3\pi \geq 0,
\tag{49}
$$

where C_i is the number of colors and t_{3LR} are the t_3 quantum numbers. A fourth family of degenerate fermions would yield $\frac{2}{3\pi} \sim 0.21$, while QCD-like technicolor models, which typically have many particles, can give larger contributions. For example, $S_{\text{new}} \sim 0.45$ from an isodoublet of fermions with four technicolors, and an entire technigeneration would yield 1.62 [37]. Non-QCD-like theories such as those involving walking could yield smaller or even negative contributions [38]. Nondegenerate scalars or fermions can contribute to S_{new} with either sign [39]. (Note that S, T, and U are induced by loop corrections and have a factor of α extracted, so they are expected to be $O(1)$ if there is new physics.)

The T parameter is analogous to ρ_0^{loop}. For a non-degenerate family

$$
T_{\text{new}} \sim \frac{\rho_0^{\text{loop}}}{\alpha} \sim 0.42 \frac{\Delta m^2}{(100 \; GeV)^2},
\tag{50}
$$

where

$$\Delta m^2 = \sum_i \frac{C_i}{3} F\left(m_{1i}, m_{2i}\right) \geq \sum_i \frac{C_i}{3}\left(m_{1i} - m_{2i}\right)^2. \tag{51}$$

Usually $T_{\text{new}} > 0$, although there may be exceptions for theories with Majorana fermions or additional Higgs doublets. In practice, higher-dimensional Higgs multiplets could mimic T_{new} with either sign (see equation (42)), and cannot be separated from loop effects unless they are seen directly or have other effects. Usually U_{new} is small.

There is enough data to simultaneously determine the new physics contributions to S, T, and U, the standard model parameters, and also $\delta_{bb}^{\text{new}} = \frac{\Gamma(b\bar{b})}{\Gamma^{\text{SM}}(b\bar{b})} - 1$. For example, S_{new}, T_{new}, U_{new}, δ_{bb}^{new}, $\hat{s}_Z^2$, $\alpha_s(M_Z)$ and m_t are constrained by M_Z, Γ, M_W, R_b, asymmetries, R, and m_t (CDF), respectively. One obtains

$$
\begin{aligned}
S_{\text{new}} &= -0.15 \pm 0.25^{-0.08}_{+0.17} & \hat{s}_Z^2 &= 0.2314(4) \\
T_{\text{new}} &= -0.08 \pm 0.32^{+0.18}_{-0.11} & \alpha_s(M_Z) &= 0.103(11) \\
U_{\text{new}} &= -0.56 \pm 0.61 & m_t &= 175 \pm 16 \text{ GeV} \\
\delta_{bb}^{\text{new}} &= 0.031 \pm 0.014,
\end{aligned}
\tag{52}
$$

where the second error is from M_H. The T_{new} value corresponds to $\rho_0 = 0.9994 \pm 0.0023^{+0.0013}_{-0.0008}$, which differs from the value in (47) because of the presence of S_{new}, U_{new}, and δ_{bb}^{new}. The data is consistent with the standard model: S_{new} and T_{new} are close to zero with small errors, and the tendency to find $S < 0$ that existed in earlier data is no longer present. The constraints on S_{new} are a problem for those classes of new physics such as technicolor which tend to give S_{new} large and positive, and S_{new} allows, at most, one additional family of ordinary fermions at 90% CL. (Of course the invisible Z width precludes any new families unless the additional neutrinos are heavier than $M_Z/2$.) The allowed regions in S'_{new} vs T'_{new} (which include the very small Higgs contributions) are shown in Figure 5. The seven parameter fit still favors a non-zero $Z \to b\bar{b}$ vertex correction δ_{bb}^{new}.

The value of $\hat{s}_Z^2$ is slightly lower than the standard model value (0.2317(3)(2)). However, the extracted $\alpha_s(M_Z)$ is considerably lower than the standard model value (0.124(5)(2)). This is entirely due to the presence of δ_{bb}^{new}. By allowing $\delta_{bb}^{\text{new}} > 0$ one can describe $R = \Gamma(\text{had})/\Gamma(\ell\bar{\ell})$ with a smaller QCD correction to $\Gamma(\text{had})$. Thus, $\alpha_s(M_Z)$ from the lineshape, though very clean in the standard model, is more sensitive to certain types of new physics than most other determinations.

7 Conclusions

- The precision data have confirmed the standard electroweak model. However, there are possible hints of discrepancies at the $2-3$ σ level in $\Gamma(b\bar{b})/\Gamma(\text{had})$ and A_{LR}^0.

- The data not only probes the tree-level structure, but the electroweak loops have been observed at the 2σ level. These consist of much larger fermionic pieces involving the top quark and QED, which only partially cancel the bosonic loops. The bosonic loops, which probe non-abelian vertices and gauge-Higgs vertices, are definitely needed to describe the data.

Figure 5: Constraints on S'_{new} and T'_{new} from various observables and from the global fit to all data, where $S'_{\text{new}} = S_{\text{new}} + S_{M_H}$, and similarly for T'_{new}. The circle, square, and diamond represent the standard model expectations for $M_H = 60$, 300, and 1000, respectively.

- The global fit to the data within the standard model yields

$$
\begin{aligned}
\overline{MS} : \hat{s}_Z^2 &= 0.2317(3)(2) & m_t &= 173 \pm 11\,^{+17}_{-19} \\
\text{on} - \text{shell} : s_W^2 &\equiv 1 - \frac{M_W^2}{M_Z^2} = 0.2242(12) & \alpha_s(M_Z) &= 0.124(5)(2),
\end{aligned}
\tag{53}
$$

where the second uncertainty is from M_H. The prediction for m_t is in remarkable agreement with the value $m_t = 174 \pm 16$ suggested by the CDF events. The data has also allowed, for the first time, a clean and precise extraction of α_s from the lineshape. This is in excellent agreement with the value $\alpha_s(M_Z) = 0.123 \pm 0.005$ from event shapes. Both are larger than many of the low energy determinations when extrapolated to the Z-pole. The lineshape determination, however, is sensitive to the presence of certain types of new physics.

- The agreement between the indirect prediction for m_t with the tentative direct CDF observation and of α_s with the various other determinations is an impressive success for the entire program of precision observables.

- Combining the direct CDF value of m_t with the indirect constraints does not make a large difference within the context of the standard model. However, when one goes beyond the standard model, the direct m_t allows a clean extraction of the new physics contributions to ρ_0, which is now shown to be very close to unity, $\rho_0 = 1.0009(18)(17)$. This strongly limits Higgs triplet vacuum expectation values and non-degenerate heavy multiplets. Similarly, it allows an extraction of the new physics contributions to S_{new}, T_{new}, U_{new}, which are consistent with zero. Finally, one can determine the new physics contributions to the $b\bar{b}$ vertex: $\delta_{b\bar{b}}^{\text{new}}$ is approximately 2.2σ away from zero, reflecting the large value of the $b\bar{b}$ width.

- The data exhibit a slight preference for a light Higgs, but this is not very compelling statistically. One finds only $M_H \leq 780(1160)$ GeV at 90(95%) CL. Furthermore, the preference depends crucially on the large observed value of $\Gamma(b\bar{b})$, and to a lesser extent on the large SLD value for A_{LR}^0. Omitting these values the M_H dependence of the observables is weak.

- The major prediction of supersymmetry is that one does not expect large deviations in the precision observables. The new particles tend to be heavy and decouple. One implication that is relevant, however, is that supersymmetric theories have a light standard model-like Higgs. They therefore favor the lighter Higgs mass and the lower end of the predicted m_t range. Also, the observed gauge couplings are consistent with the coupling constant unification expected in supersymmetric grand unification, but not with the simplest version of non-supersymmetric unification.

- In compositeness and dynamical symmetry breaking theories one typically expects not only large flavor changing neutral currents but significant deviations of ρ_0 from unity and of S_{new} and T_{new} from zero. One further expects that $\delta_{bb}^{\mathrm{new}} < 0$, at least in the simplest models. Therefore, the precision experiments are a major difficulty for this class of models.

Acknowledgement

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